Name: KEY

Relativity Problems IV: *Energy & Momentum*

1. a. What is the rest energy of an electron in J?

$E_0 = mc^2 = (9.1\times10^{-31})(3\times10^8)^2 = \boxed{8.2\times10^{-14}\ J}$

b. What is the rest energy of an electron in eV?

$(8.2\times10^{-14}\ J)(1\ eV / 1.6\times10^{-19}\ J) = \boxed{510{,}000\ eV}$

c. What is the mass of a muon, which has a rest energy of 106 MeV?

$E_0 = mc^2 \quad (106\times10^6\ eV)(1.6\times10^{-19}\ J/eV) = m(3\times10^8)^2 \quad \boxed{m = 1.88\times10^{-28}\ kg}$

d. In this unit, what do we mean when we talk about the total energy of a particle?

$E = K + E_0$ → just kinetic + rest

2. a. What is the kinetic energy of an electron that has a speed of 2.5×10^8 m/s?

$K = (\gamma - 1)mc^2 \quad \gamma = \frac{1}{\sqrt{1-\left(\frac{2.5}{3}\right)^2}} \quad \gamma = 1.809 \quad K = (1.809-1)(9.1\times10^{-31})(3\times10^8)^2$

$\boxed{K = 6.6\times10^{-14}\ J}$

b. What is the momentum of an electron that has a speed of 2.5×10^8 m/s?

$p = \gamma m v = (1.809)(9.1\times10^{-31})(2.5\times10^8) \quad \boxed{p = 4.1\times10^{-22}\ kg\cdot m/s}$

3. A particle has a total energy of 2.0 MeV and a rest energy of 0.75 MeV. How fast is it going?

$E = 2$ MeV
$E_0 = 0.75$ MeV

$E = \gamma E_0$ ← Note: the units of E & E_0 cancel! So just keep them MeV!

$2 = \gamma(.75)$

$\gamma = 2.667$ →

$2.667 = \frac{1}{\sqrt{1-\beta^2}}$

$7.111(1-\beta^2) = 1$

$\beta^2 = \frac{6.111}{7.111} = .86$

$\boxed{\beta = 0.93}$

4. How fast is an electron moving that has a total energy of 1.5 MeV?

from 1b, $E_0 = .51$ MeV

So $E = \gamma E_0 \quad 1.5 = \gamma(.51) \quad \gamma = 2.94$ →

$2.94 = \frac{1}{\sqrt{1-\beta^2}}$

$\beta^2 = \frac{7.65}{8.65} = .88$

$\boxed{\beta = 0.94}$

Relativity Problems IV: *Energy & Momentum*

5. What is the kinetic energy of a particle moving at speed 0.9c and with a rest energy of 25 MeV?

$\beta = 0.9$

$\gamma = \frac{1}{\sqrt{1-.9^2}} = 2.29$

$K = (\gamma - 1)E_0$

$= (2.29 - 1)(25)$

$K = 32.4$ MeV

6. How much work would it take to get a proton with a rest energy of 940 MeV to a speed of 0.95c?

Since $W = \Delta K$, just find K!

$\beta = 0.95 \rightarrow \gamma = \frac{1}{\sqrt{1-.95^2}} = 3.20$

$K = (\gamma - 1)E_0$

$K = (3.2 - 1)(940) = 2070$ MeV

7. If the kinetic energy of a particle is equal to its rest energy, what is its speed?

$K = E_0$ & $K = (\gamma - 1)E_0$

so $E_0 = (\gamma - 1)E_0$

$1 = \gamma - 1 \rightarrow \gamma = 2$

$2 = \frac{1}{\sqrt{1-\beta^2}}$ $\beta^2 = \frac{3}{4}$

$\beta = 0.87$

8. A particle of mass m has a momentum equal to mc. What is the speed of the particle?

$p = \gamma m v$

So $mc = \gamma m v$

$mc = \gamma m \beta c$

$1 = \gamma \beta$

$1 = \frac{\beta}{\sqrt{1-\beta^2}}$

$(1-\beta^2) = \beta^2$

$2\beta^2 = 1$

$\beta^2 = \frac{1}{2}$

$\beta = 0.71$

Please note that for the following problems, use six decimal points for the masses of accuracy for the masses. (You can still use c = 3 x 10⁸ m/s if you like, or you can be particular and use c = 2.997925 x 10⁸ m/s.)

9. When dealing with particles, masses are often given in "atomic mass units", or "u." By definition, 1 u is 1/12 the mass of the carbon-12 atom. What is the energy equivalent of 1 u

a. in Joules?

$E_0 = mc^2 = (1.66054 \times 10^{-27})(3 \times 10^8)^2 = 1.49 \times 10^{-10}$ J

b. in MeV?

$\frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} = 934{,}000{,}000$ eV $= 934$ MeV

Relativity Problems IV: *Energy & Momentum*

10. Deuteron, a hydrogen isotope, is one of the simplest nuclei and is made of 1 proton and 1 neutron. The mass of a free proton is 1.00728u, the mass of a free neutron is 1.00866u. The mass of the deuteron is 2.01355u.
    a. Why is the mass of the deuteron less than the sum of the individual particles?

    Because energy is released when they combine

    b. How much energy would be released by combining a proton and neutron?

    The mass "lost" is:

    $1.00728 + 1.00866 - 2.01355$

    $= .00239\ u$

    So $(.00239\ u)(934\ \frac{MeV}{u})$

    $= 2.23\ MeV$

    c. How much energy would it take to rip apart a deuteron into a proton and a neutron?

    $2.23\ MeV$

    If that is the energy released when they combine, you need that much energy to rip them apart!

11. A U-236 atom is very unstable and decays according to the following reaction:

$$U_{92}^{236} \rightarrow Ba_{56}^{144} + Kr_{36}^{89} + 3n$$

The masses are: U-236 = 236.045568 u; Ba-144 = 143.922953 u; Kr-89 = 88.91763 u. How much energy is released in this reaction?

The mass lost is

$236.015568 - (143.922953 + 88.91763 + 3(1.00866))$

$= 0.179005\ u$

So $(.179005)(934) = 167\ MeV$

12. Two deuterons are combined to form an helium nucleus. If 23.8 MeV of energy is released in this process, what is the mass of the He nucleus?

The mass lost would be

$\frac{23.8\ MeV}{934\ MeV/u} = 0.025482\ u$

2 Deuterons are

$2(2.01355) = 4.02710\ u$

The He Nucleus will be less mass b/c energy was released:

$4.02710 - .025482$

$= 4.01618\ u$

Relativity Problems IV: *Energy & Momentum*

Answers:

1 a) 8.2×10^{-14} J	*b) 510 keV*	*c) 1.88×10^{-28} kg*	*d) rest energy + kinetic energy*
2 a) 6.6×10^{-14} J	*b) 4.1×10^{-22} kgm/s*	*3) 0.93c*	*4) 0.94c*
5) 32 MeV	*6) 2000 MeV*	*7) 0.87c*	*8) 0.71c*
9. a) 1.49×10^{-10} J	*b) 934 MeV*	*10. b) 2.23 MeV*	*c) the same as b!*
11) 167 MeV	*12) 4.001618 u*		